Name: Mahyar Pirayesh Date: Oct. 9th, 2023

Math 12 Enriched: HW Section 3.1 Graphing Polynomial Functions

1. Indicate which of the following are polynomials. Circle them and state the degree:

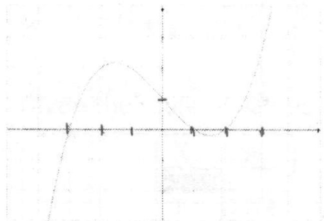
a) $y=\sqrt{3x^2}-2x+5$ NO $=\sqrt{3}\lvert x\rvert-2x+5$	b) $y=\sqrt{3}x^2-4x+5$ YES 2nd degree	c) $y=10$ YES 0th degree	d) $y=2^x$ NO
e) $y=(x-3)^2$ YES 2nd degree	f) $y=2x$ YES 1st degree	g) $\dfrac{2x^2-3x+5}{10}$ YES 2nd degree	h) $y=\dfrac{2x^2-3x+5}{2x}$ NO
i) $y=\dfrac{1}{2x^2-3}$ NO	j) $y=\sqrt{3x^4}-3x$ YES 2nd degree $=\sqrt{3}\lvert x\rvert^2-3x$ $=\sqrt{3}\,x^2-3x$	k) $y=(x-5)^{-1}$ $=\dfrac{1}{x-5}$ NO	L) $y=\dfrac{x^2-4}{x+2}$ NO Non-continuous, so NO. $x\neq 2$

2. Indicate the degree and the number of roots for the following equations:

a) $y=(x-3)(x+4)(2x-1)$ Degree: 3 # of Roots: 3 $r_1=3$, $r_2=-4$, $r_3=\frac{1}{2}$	b) $y=(x^2-4)(x^2-1)$ $y=(x-2)(x+2)(x-1)(x+1)$ Degree: 4 # of Roots: 4	c) $y=-x(x^2-3)(x^2+1)$ $-x(x-\sqrt{3})(x+\sqrt{3})(x^2+1)$ Degree: 5 # of Roots: 3 real roots
d) $y=(x^2+1)(x^4+9)$ Degree: 6 # of Roots: No real roots (only imaginary)	e) $y=x^4+4x^3+6x^2+4x+1$ Pascal's triangle! $y=(x+1)^4$ Degree: 4 # of Roots: 1	f) $y=x^3+2x^2-5x-6$ Degree: 3 # of Roots: 3

3. State the roots, y-intercepts, domain, range, and the equation in factored form.

I



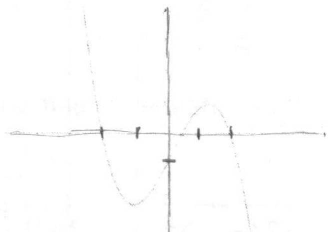
Roots: $(-3,0)$, $(2,0)$, $(1,0)$ Degree: 3

Equation in Factored Form:

$y=\frac{1}{6}(x+3)(x-2)(x-1)$

should be $\frac{1}{6}$ so that y-int. ends up as 1. $\frac{1}{6}\cdot 3\cdot -2\cdot -1=1$ ✓

ii)



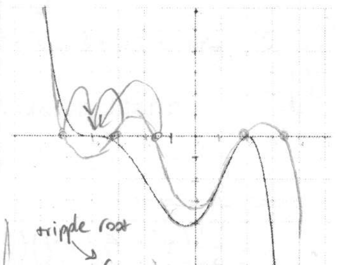
Roots: $(-2,0)$, $(\frac{1}{2},0)$, $(1,0)$ Degree: 3

Equation in Factored Form

$y=-(x+2)(x-\frac{1}{2})(x-1)$

y-int $=2(-\frac{1}{2})(-1)=-1$ ✓

iii)



Roots: $(-4,0)$, $(2,0)$ Degree: 5

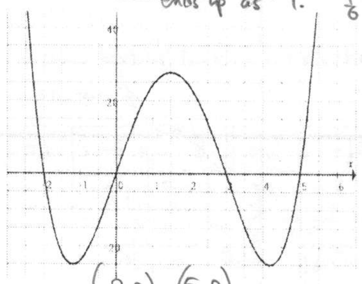
tripple root

double root

Equation in Factored Form

$y=\frac{1}{64}(x+4)^3(x-2)^2$

iv)

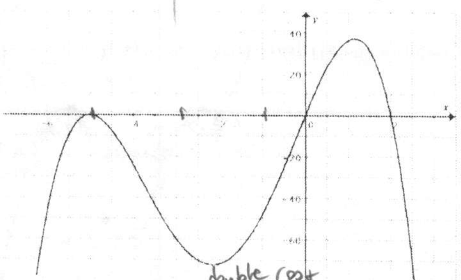


Roots: $(-2,0)$, $(0,0)$, $(3,0)$, $(5,0)$ Degree: 4

Equation in Factored Form:

$y=x(x+2)(x-3)(x-5)$

d)



Roots: $(-5,0)$, $(0,0)$, $(2,0)$, Degree: 4

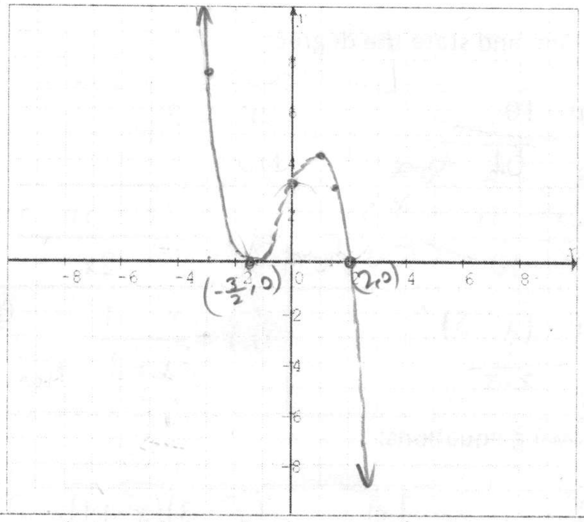
double root

Equation in Factored Form:

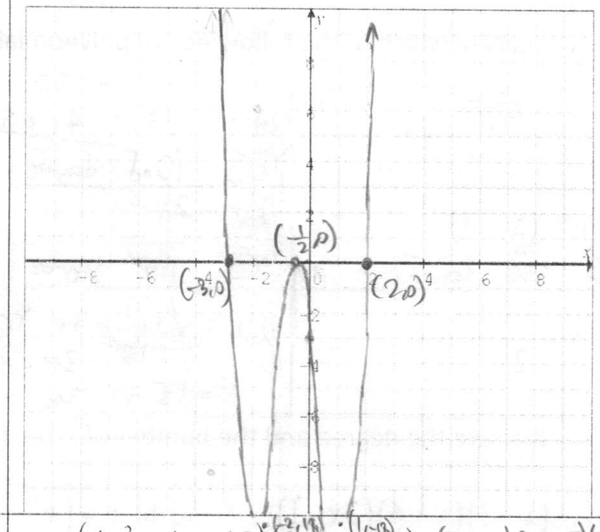
$y=-x(x+5)^2(x-2)$

4. Given each of the following equations in factored form, graph it on the grid provided.

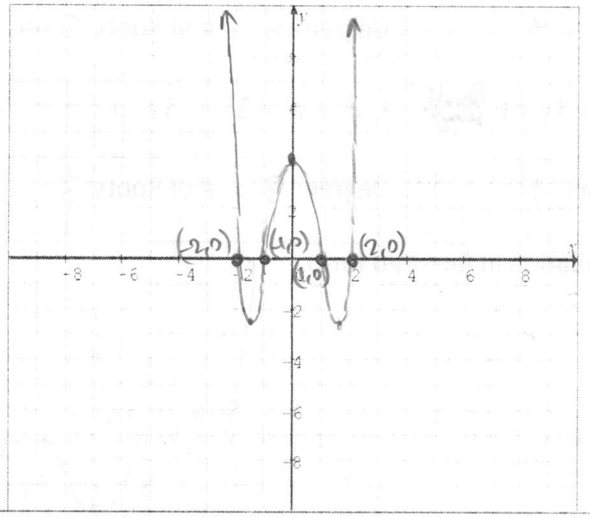
a) $y=-\frac{1}{6}(x-2)(2x+3)^2$



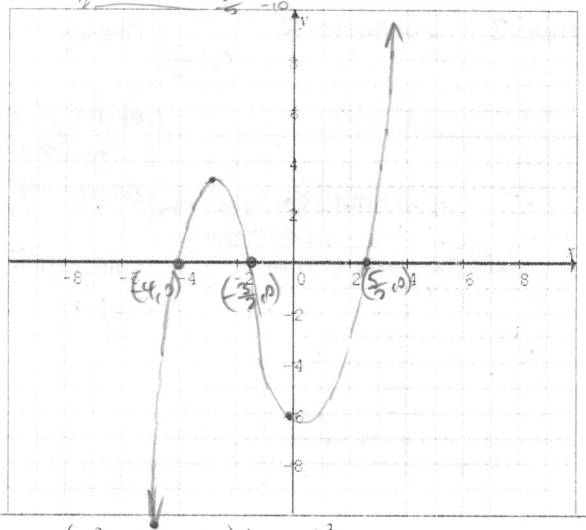
b) $y=\frac{1}{2}(x-2)(x+3)(2x+1)^2$



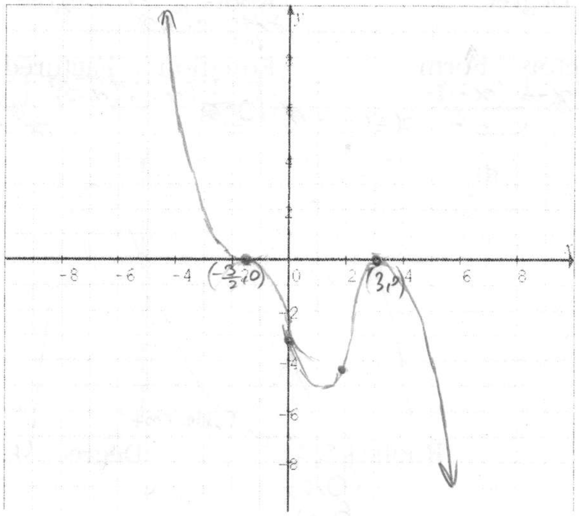
$y=(x^2-1)(x^2-4)=(x-1)(x+1)(x-2)(x+2)$



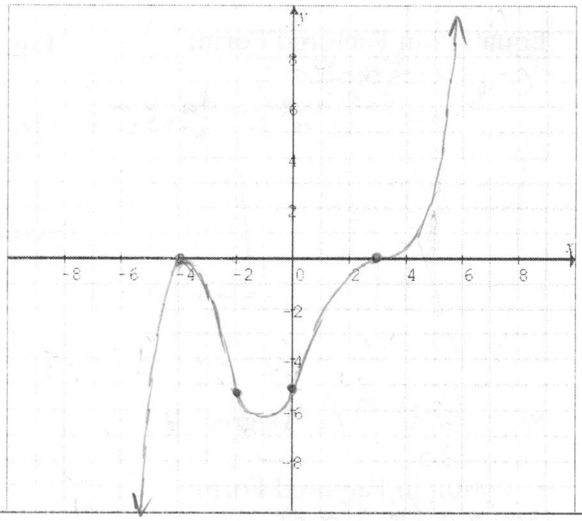
$y=(4x^2-4x-15)(0.1x+0.4)=(2x+3)(2x-5)(0.1x+0.4)$



$y=\frac{-1}{81}(x^2-6x+9)(2x+3)^3=-\frac{1}{81}(x-3)^2(2x+3)^3$



$y=\frac{(x^2+8x+16)(x-3)^3}{96}=\frac{1}{96}(x+4)^2(x-3)^3$



5. Indicate whether of the following statements are either true of false

   a. The domain of all polynomial functions is all real numbers [✓ TRUE] / FALSE

   what abt- $x=4$? not a polynomial?

   b. The range of all polynomial functions is all real numbers TRUE / [✓ FALSE]

   c. The range of $y = Ax^2 - Bx^3 + C \quad (A, B, C \neq 0)$ is all real numbers [✓ TRUE] / FALSE

   d. The degree of the following polynomial function is 5

      i. $y = x\left(x^2 - 1\right)\left(x^2 + 1\right)$ [✓ TRUE] / FALSE

      ii. $y = x\left(2x^2 - 3x + 6x^3 + 3x\right)$ TRUE / [✓ FALSE]

      iii. $y = (2x-4)\left(2x^3 - 4x + 4x^2\right)(4x - x)$ [✓ TRUE] / FALSE

      iv. $y = (2x-4)\left(3 - 3x - 2x^2\right)(3x - 3x)(5x - 7x)$ TRUE / [✓ FALSE]

6. The polynomial $P(x) = x^3 + ax^2 + bx + c$ has the property that the mean of its zeros, the product of its zeros, and the sum of its coefficients are all equal. If the y-intercept of the graph of $y = P(x)$ is 2, what is the value of "b"?

$\frac{r_1+r_2+r_3}{3} = r_1r_2r_3 = 1+a+b+c \quad ; \quad c=2$

Viera Sums:

$-a = r_1+r_2+r_3 \Rightarrow \frac{-a}{3} = \frac{r_1+r_2+r_3}{3}$

$b = r_1r_2 + r_1r_3 + r_2r_3$

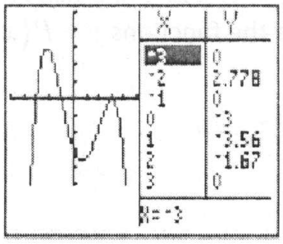
$-c = r_1r_2r_3 \Rightarrow -2 = r_1r_2r_3$

$\Rightarrow \underbrace{-\frac{a}{3} = -2}_{a=6} = 3+a+b$

$-2 = 3+a+b \Rightarrow -2 = 3+6+b$

$\Rightarrow \boxed{b=-11}$

7. Given the table of values and graph below, find the equation of the polynomial in factored form:



X	Y
-3	0
-2	2.778
-1	0
0	-3
1	-3.56
2	-1.67
3	0

X=-3

Roots: $-3, -1, 3$

Y-intercept: $(0,-3)$

$y = a(x+3)(x+1)(x-3)$

constant $= 3 \cdot 1 \cdot (-3) \cdot a = -9a$

$\therefore a = \frac{1}{3}$

$$\boxed{y = \frac{1}{3}(x+3)(x+1)(x-3)}$$

8. A polynomial function has the following table of values. Find the equation of the polynomial using finite difference

$y = ax^4 + bx^3 + cx^2 + dx + e$

x	-4	-3	-2	-1	0	1	2
y	-21	30.5	35	22.5	11	6.5	3

$\leftarrow L_1$

$\leftarrow L_2$

51.5 4.5 -12.5 -11.5 -4.5 -3.5

-47 -17 +1 7 1

+30 18 6 -6

-12 -12 -12

STAT + QuarticReg:

$a = -0.5$

$b = 1.1 \times 10^{-12} \approx 0$

$c = 4$

$d = -8$

$e = 11$

if k=4, no real roots

9. If $y = x^4 + kx^2 + 4$ has 2 pairs of repeated roots only, find all the possible values of "k".

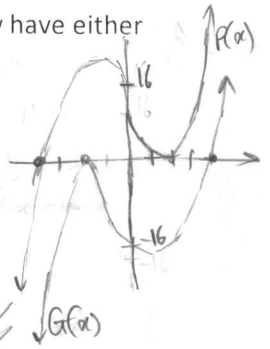
$x^2 = \frac{-k \pm \sqrt{k^2-16}}{2}$ $\quad x = \pm\sqrt{\frac{-k\pm\sqrt{k^2-16}}{2}} \Rightarrow k^2-16=0 \Rightarrow k = \cancel{4}, -4$ $\boxed{k=-4}$

If $\Delta = 0$, then there will be 2 roots only

10. Both equations $x^3 - 12x + 16 = 0$ and $x^3 - 12x - 16 = 0$ have a double root and one other root that is different from the double root. Use this information to determine which of the equations below have either i) 3 different roots OR ii) only ONE root.

$P(x) = x^3 - 12x + 16 = (x+4)(x-2)^2$

$G(x) = x^3 - 12x - 16 = (x+2)^2(x-4)$



$\boxed{\text{1 ROOT}}$

a) $x^3 - 12x + 20 = 0$

P(x) is shifted up by 4

b) $x^3 - 12x + 10 = 0$

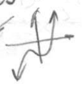
P(x) shifted down by 6

$\boxed{\text{3 ROOTS}}$

c) $x^3 - 12x - 20 = 0$

G(x) shifted 4 down

$\boxed{\text{1 ROOT}}$



11. Determine the values of "k" for which the equation $x^3 - 12x + k = 0$ will have:

i) 3 different roots

$\boxed{-16 < k < 16}$

ii) 2 different roots

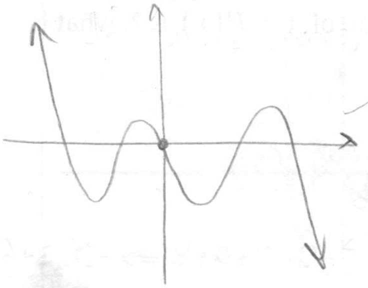
$\boxed{k \in \{-16, 16\}}$

iii) only one root

$\boxed{k > 16 \text{ or } k < -16}$

We can shift P(x) down anywhere between P(x) and G(x).

12. The graph of the polynomial $P(x) = x^5 + ax^4 + bx^3 + cx^2 + dx + e$ has five distinct x-intercepts, one of which is at (0,0). Which of the following coefficients cannot be zero? a? b? c? d? e?



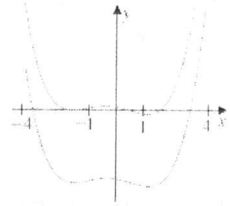
Since our y-intercept is at (0,0), e = 0.

$P(x) = x^5 + ax^4 + bx^3 + cx^2 + dx = x(ax^4 + bx^3 + cx^2 + dx + e)$

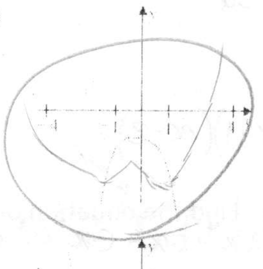
If e=0 then the equation could be factored to get you $P(x) = x^2(ax^3 + bx^2 + cx + d)$ which has 4 distinct roots with a double root at 0. Therefore, $\boxed{d \neq 0}$

13. Challenge: The nonzero coefficients of the polynomial P(x) with real coefficient are all replaced by their mean to form another polynomial Q(x). Which of the following graphs below can be the functions $y = P(x)$ and $y = Q(x)$ over the interval $-4 \le x \le 4$? amc12 2002 #25

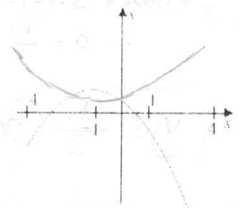
(A)



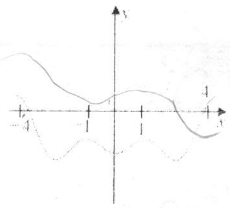
(B)



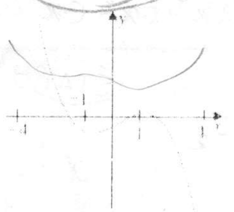
(C)



(D)



(E)



$P(x) = ax^4 + bx^3 + cx^2 + dx + e$

$P(1) = a + b + c + d + e$

Even if we replace every coefficient with the average, P(1) will still remain the same

Only in (B) does (1,y) stay the same, so (B) is the only solution!

If $p(x)$ is a cubic polynomial with $p(1) = 1$, $p(2) = 2$, $p(3) = 3$, $p(4) = 5$, find $p(6)$.

$$\begin{array}{c|c|c|c|c} x & y & & & \\ \hline 1 & 1 & & & \\ & & 1 & & \\ 2 & 2 & & 0 & \\ & & 1 & & 1 \\ 3 & 3 & & 1 & \\ & & 2 & & \\ 4 & 5 & & & \\ & & & & \\ 6 & ? & & & \end{array}$$

$$a+b+c=1$$
$$8a+4b+2c=2$$
$$27a+9b+3c=3$$
$$64a+16b+4c=5$$

$$7a+3b+c=1$$
$$19a+5b+c=1$$
$$37a+7b+c=2$$

$$12a+2b=0$$
$$18a+2b=1$$

$$6a=1 \Rightarrow a=\frac{1}{6}$$
$$b=-1$$
$$c=-\frac{17}{6}$$

$$P(6)=\frac{1}{6}(6)^3-(6)^2+\frac{17}{6}=-\boxed{\frac{17}{6}}$$

d

(1977 AHSME #21) For how many values of the coefficient a do the equations ~~have~~ have a common real solution?

$$0 = x^2+ax+1 \text{ and } 0 = x^2-x-a$$

Roots: r_1, r_2 Roots: r_1, r_3

$$1 = r_1r_2 \Rightarrow r_2=\frac{1}{r_1} \qquad -a = r_1r_3$$

$$-a = r_1+r_2 \qquad 1 = r_1+r_3 \Rightarrow r_3 = r_1-1$$

$$-a = r_1+\frac{1}{r_1} \qquad -a = r_1(1-r_1)$$

$$\cancel{r_1}+\frac{1}{r_1} = \cancel{r_1}-r_1^2$$

$$1 = -r_1^3$$

$$\underline{r_1=-1}$$

$$-a = r_1+\frac{1}{r_1} \Rightarrow -a = -1-1 \Rightarrow \boxed{a=2}$$